

NPS ARCHIVE
1962
WILLIAMS, F.

COMPUTATION OF HEMISPHERIC ENERGY
CONVERSION IN THE ATMOSPHERE UTILIZING
VERTICAL MOTIONS AT FIVE LEVELS

FORREST R. WILLIAMS

LIBRARY
U.S. NAVAL POSTGRADUATE SCHOOL
MONTEREY, CALIFORNIA





COMPUTATION OF HEMISPHERIC ENERGY CONVERSION
IN THE ATMOSPHERE UTILIZING
VERTICAL MOTIONS AT FIVE LEVELS

* * * * *

Forrest R. Williams

COMPUTATION OF HEMISPHERIC ENERGY CONVERSION
IN THE ATMOSPHERE UTILIZING
VERTICAL MOTIONS AT FIVE LEVELS

by

Forrest R. Williams
Lieutenant, United States Navy

Submitted in partial fulfillment of
the requirements for the degree of

MASTER OF SCIENCE
IN
METEOROLOGY

United States Naval Postgraduate School
Monterey, California

1 9 6 2

02
02
ILLIAMS, R.

148213
~~U5476~~

COMPUTATION OF HEMISPHERIC ENERGY CONVERSION
IN THE ATMOSPHERE UTILIZING
VERTICAL MOTIONS AT FIVE LEVELS

by

Forrest R. Williams

This work is accepted as fulfilling
the thesis requirements for the degree of

MASTER OF SCIENCE

IN

METEOROLOGY

from the

United States Naval Postgraduate School

ABSTRACT

By programming the Control Data Corporation 1604 Computer, computations of conversion between available potential energy and kinetic energy in the atmosphere over the Northern Hemisphere are made for 21 January 1962. Due to the recent availability of vertical motion at five levels, the energy conversions were calculated for the 300-mb, 500-mb, 700-mb, 850-mb, and 1000-mb pressure surfaces. The computations were subdivided into conversion due to mean sinking or lifting of the air mass, zonal conversion, and eddy conversion. The relative contribution of 21 latitude bands to the eddy conversion is also presented along with the spectral components of eddy conversion for waves 1 through 15.

Additionally, the meridional circulation indicated by the zonally averaged omega at the five surfaces on 21 January 1962 is given. Comparison is then made with earlier research which used vertical motion only at the 600-mb surface.

The writer wishes to express his appreciation to Professor George J. Haltiner of the U. S. Naval Postgraduate School for his assistance and guidance in this investigation.

Appreciation is also expressed to Mr. Milton H. Reese of the U. S. Navy Fleet Numerical Weather Facility for his assistance in programming.

TABLE OF CONTENTS

Section	Title	Page
1.	Introduction	1
2.	Background	2
3.	Procedures	4
4.	Results of computations	10
5.	Conclusions	11
6.	Bibliography	14

Table

1.	Energy conversion in the atmosphere north of 21N for 1200Z, 21 January 1962, in units of 10^{20} ergs $\text{mb}^{-1} \text{sec}^{-1}$	10
----	--	----

LIST OF ILLUSTRATIONS

Figure		Page
1.	Zonal averages of omega for five pressure surfaces at 1200Z, 21 January 1962	7
2.	Negative cospectrum of omega and temperature, $-[\omega' T']$, for five surfaces at 1200Z, 21 January 1962	12
3.	Spectral components of eddy conversion for five surfaces at 1200Z, 21 January 1962	13

TABLE OF SYMBOLS

λ	longitude
ϕ	latitude
p	pressure
a	radius of the earth
n	wave number around a latitude band
t	time
ω	dp/dt = individual pressure change
T	temperature (degrees Absolute)
g	the acceleration of gravity
R	gas constant
$[x]$	$\frac{1}{2\pi} \int_0^{2\pi} x d\lambda =$ zonal average of x
x'	$x - [x] =$ deviation from the zonal average
$\{x\}$	$\frac{1}{\sin\phi_2 - \sin\phi_1} \int_{\phi_1}^{\phi_2} x \sin\phi d\phi =$ meridional average of x between latitude ϕ_1 and ϕ_2
x''	$x - \{x\} =$ deviation from the meridional average
\overline{x}	area average over a constant-pressure surface

1. Introduction

The conversion between available potential energy and kinetic energy in the atmosphere has been computed by Wiin-Nielsen [1] and by Saltzman and Fleisher [2]. Both of these studies of hemispheric energy conversion used the vertical motion field at the 600-mb pressure surface from the Joint Numerical Weather Prediction Unit (JNWPU). The model through which JNWPU computed vertical motion was a simple 2-parameter baroclinic prediction scheme which assumed that flow in the atmosphere is quasi-geostrophic, frictionless, and adiabatic; consequently, the vertical motions must be treated with caution. As emphasized by Saltzman, his measurements apply only to a single pressure surface in the middle troposphere. If these computations are to be accepted as a fair estimate of the rate of conversion for the entire atmosphere, it must be assumed that the events occurring at all levels, weighted by mass and integrated, are proportional to the results as obtained at the 600-mb surface.

In the spring of 1962, vertical motions at five surfaces became available from a model constructed at the U. S. Naval Postgraduate School. By solving the ω -equation, Clarke and Lawniczak [3] computed vertical motions in the atmosphere at 300 mb, 500 mb, 700 mb, 850 mb, and the lower boundary. Their model is also quasi-geostrophic, adiabatic, and basically frictionless; however, the vertical motion at the lower boundary includes the effects of surface friction and mountainous terrain, as opposed to the lower boundary condition of zero vertical motion found in the JNWPU model used in the two previously mentioned investigations.

The purpose of this investigation is to compute energy conversion at these five surfaces and to compare the results with those using only vertical motion at the 600-mb pressure surface. Additionally, the

zonally averaged meridional circulation at the five surfaces will be studied and compared to that found earlier at 600 mb.

2. Background

As given by Lorenz [4], the total rate of conversion from available potential energy to kinetic energy per unit area and per unit pressure difference may be written in the form

$$C = - \frac{R}{f_0} \left\{ \overline{u \cdot \nabla \cdot \mathbf{v}} \right\} \quad (1)$$

where the area average is taken over a closed pressure surface. The total conversion was expanded by Saltzman into the following subdivisions:

$$C = C_0 + C_2 + C_E, \quad (2)$$

where

$$C_0 = - \frac{R}{f_0} \left\{ \overline{u \cdot \nabla \cdot \mathbf{v}} \right\} \quad (3)$$

is the rate of conversion from available potential energy to kinetic energy due to mean vertical lifting or sinking of the air mass;

$$C_2 = - \frac{R}{f_0} \left\{ \overline{u \cdot \nabla \cdot \mathbf{v}} \right\} \quad (4)$$

is the rate of conversion from zonal available potential energy to zonal kinetic energy; and

$$C_E = - \frac{R}{f_0} \left\{ \overline{u \cdot \nabla \cdot \mathbf{v}} \right\} \quad (5)$$

is the rate of conversion from eddy available potential energy to eddy kinetic energy.

Inasmuch as C_0 would be zero if vertical motion over the entire earth were considered, Saltzman did not include the term C_0 in his

computations, even though \bar{u} , as computed over the limited area of his grid, 20N to 80N, conceivably might have been non-zero. Wiin Nielsen similarly did not include \bar{u} , in his measure of the total energy conversion; however, he did compute it for purposes of comparison. His \bar{u} -field was available from an octagonal grid having a southern boundary with a mean latitude of 17N.

The spectral components of \bar{u} also have been resolved in previous investigations. It has been found that, over a monthly average, eddies of wave number 6 or 7 were most active for the months of January, February, and April in 1959. This result agreed with the prediction of the linear theories of baroclinic instability that the maximum growth rate occurs for waves of intermediate scale (i.e., waves having a length of roughly 5000 to 4000 km in the middle latitudes). Spectral analysis of eddy conversion has been computed only to wave 15 since that is the shortest wave, about 1800 km in middle latitude, which could probably be analyzed in our present meteorological system. In any case, it was found that the energy conversion was negligible beyond wave number 10.

Latitudinally, Saltzman found that the greatest contribution to the eddy conversion was near 45N for February 1959. Similarly for January and April 1959, Wiin-Nielsen found that the latitude band near 45N had the greatest correlation between vertical velocity and mean temperature (i.e., total conversion from available potential energy to kinetic energy was greatest near latitude band 45N).

Both investigations found that the eddy energy conversion was roughly an order of magnitude greater than the zonal conversion. Also, if one considers the conversion due to mean lifting or sinking of the air mass over the limited area concerned (i.e., not over the entire globe),

it is found that this portion of the conversion is from kinetic energy to potential energy. At least this was found for January and April of 1959, when the mean vertical motion was downward.

Monthly averaging over the three months mentioned, further found indications of the classical three-cell meridional circulation. Rising motions to the south indicate the existence of a direct cell (i.e., a kinetic energy producing cell), followed at higher latitudes by an indirect cell, and still farther north by the weaker suggestion of another direct cell.

3. Procedures

In order to compute the conversions of energy, use was made of the Control Data Corporation Model 1604 Computer, using the octagonal grid from the U. S. Navy Fleet Numerical Weather Facility (FNWF), Monterey, California. This grid, centered on the pole and extending to approximately 10N latitude, consists of 1977 grid points with a grid size of 381 km at 60N latitude.

In programming, the total rate of conversion per mb over the entire pressure surface of the grid, rather than the total rate of conversion per mb per unit area of the surface, was computed. Therefore, the quantity sought became

$$C_p = - \frac{R}{g} \int_0^{\pi} \int_0^{2\pi} \omega T \cos \phi \, d\lambda \, d\phi \quad (6)$$

Through averaging "operators" given in the Table of Symbols, this expression was expanded into the form

$$C_p = - \frac{R}{g} \pi \alpha^2 (\overline{\omega T} - \overline{\omega} \overline{T}) \left[\left\{ \overline{\left(\frac{\omega}{\alpha} \right)^2} \right\} + \left\{ \overline{\left(\frac{T}{\alpha} \right)^2} \right\} + \left\{ \overline{\left(\frac{\omega T}{\alpha^2} \right)} \right\} \right], \quad (7)$$

where the appearance of the extra terms, not found in equations (4), (5), and (6), changes the units so as to represent the energy conversion over the total area.

The initial data consisted of the temperature fields available at the 300-mb, 500-mb, 700-mb, 850-mb, and 1000-mb pressure surfaces from the "master save" tapes of FNWF, and of the omega fields similarly available at the 300-mb, 500-mb, 700-mb, and 850-mb surfaces. However, the omega field is computed for the lower boundary at terrain height. Naturally, the latter will differ greatly from the 1000-mb pressure surface when the area of the grid under consideration is in elevated or mountainous terrain. Therefore, the conversion rates computed for pressure surfaces intersecting the lower boundary must be used with caution, since in elevated terrain they represent a fictitious conversion.

Here it will be noted that the relaxation technique used in forming the omega fields did not give values of omega for the outer two rows of grid points of the octagonal grid. The southern boundary of acceptable ω -values then became 21N, and no products of omega and temperature were accepted south of that boundary.

To form the zonal averages of either field, use was made of a program devised by G. 'Arnason and by M. H. Reese. With slight modifications necessary to permit the program to accept the scaling of the temperature and omega fields, the program produced values of the zonal averages of omega, $[\omega]$, and of temperature, $[T]$, for 21 zonal bands, each being one grid length in width. These averages were formed, therefore, for bands extending from 21N to the North Pole. This program further produced Fourier coefficients of the wave components, as will be discussed later.

Separate use was made of the output of this program, operating on

the omega fields, to determine any meridional circulations that might be indicated by the five surfaces. Figure 1 shows the zonal average of omega for the 21 bands for 21 January 1962. It should be recalled that a positive omega represents downward vertical motion. The southernmost band is centered at approximately 23N. The vertical consistency of the five surfaces appears quite good. Large vertical motions downward between 25N and 45N seem quite appropriate for the northern portion of a Hadley direct cell and for the southern portion of the indirect Ferrel cell. Although the general picture given by this analysis of one day approximates the monthly average figure found by earlier studies of the 600-mb pressure surface only, the magnitude of the vertical motion for this day was considerably greater than that of the monthly averages computed by previous studies.

Now returning to the computation of energy conversion, each level first was treated separately; specifically, the computations were made for the 1000-mb surface, followed by the four upper pressure surfaces. To illustrate the manner in which the computer program was written, the procedure will be described for only one surface.

First, $\overline{\omega}_T$, the subscript T indicating that the conversion is in units per total area of the grid, was formed. Having the 21 values of both $\overline{\omega}_T$ and \overline{v} , the meridional averages were formed of them (i.e., $\overline{\overline{\omega}_T}$ and $\overline{\overline{v}}$). The meridional average operator was formed by the finite difference formula

$$\overline{\overline{\omega}_T} = \frac{1}{\Delta\phi} \sum_{l=0}^{20} \left(\frac{\omega_T + X_l}{2} \right) \left(\sin \phi_{l+1} - \sin \phi_l \right), \quad (8)$$

where $l = 20$ is the southernmost latitude band centered at approximately 23N and $l = 0$ represents the pole. A product of the two meridional

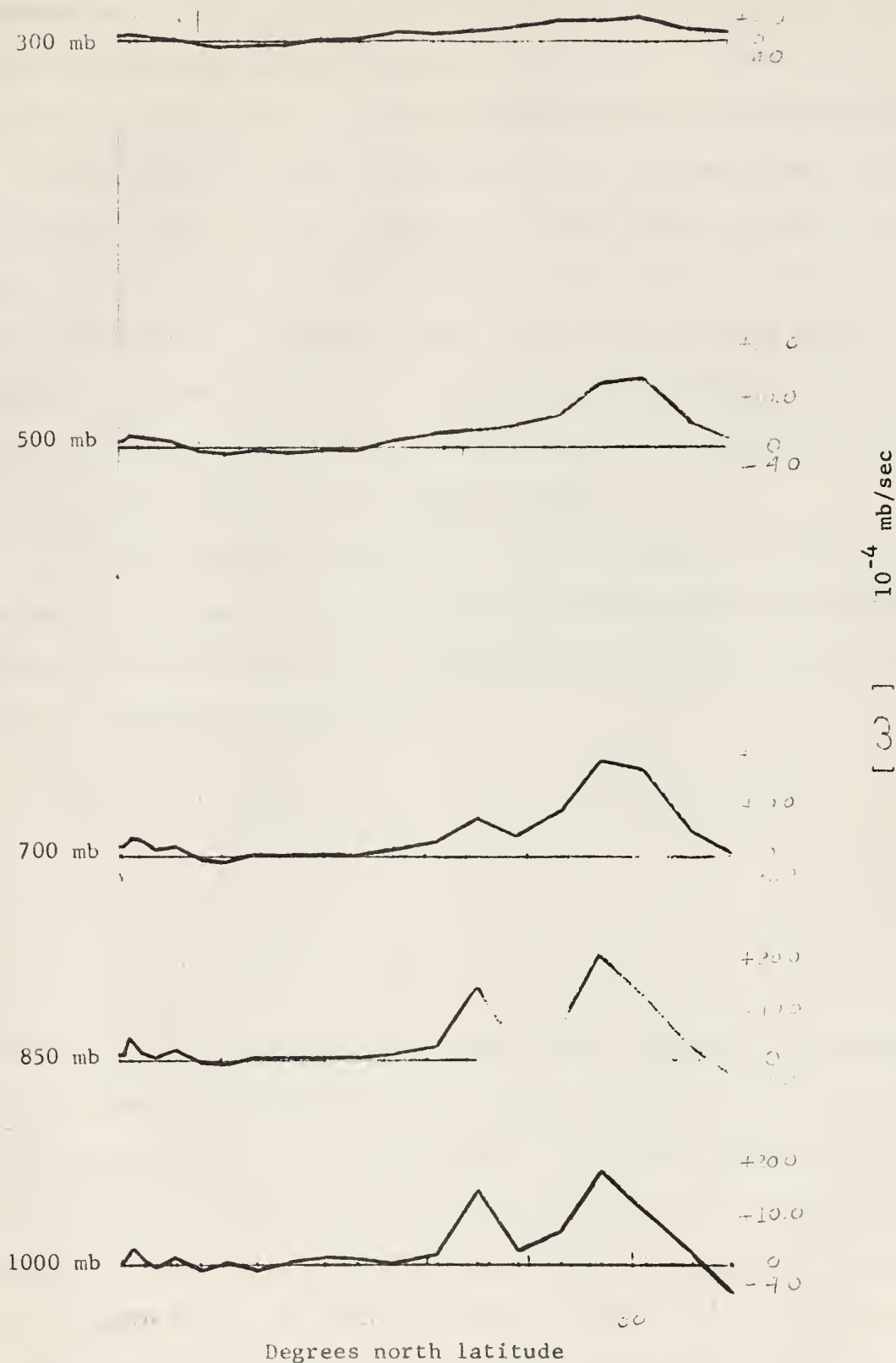


Figure 1. Zonal averages of omega for five pressure surfaces at 1200Z, 21 January 1962.

averages was formed, and then by multiplying this product by a constant representing the terms before the large bracket in equation (7), the rate of conversion between potential energy and kinetic energy per unit pressure difference over the entire area of the grid north of 21N was obtained.

Next, to form the zonal conversion, $\overline{C_{zT}}$, the meridional deviation of both $[\omega]$ and $[\tau]$ were computed for each of the 21 latitude bands (i.e., $[\omega]' = [\omega] - \{[\omega]\}$ and $[\tau]' = [\tau] - \{[\tau]\}$). Then, the products of these meridional deviations for the 21 bands were averaged, yielding $\{[\omega]' [\tau]'\}$, which, when multiplied by the constant indicated in equation (7), produced the rate of conversion from zonal potential energy to zonal kinetic energy.

To produce the eddy conversion, $\overline{C_{edT}}$, the Fourier analysis technique was used. The program referenced earlier which was obtained from FNWF to produce the zonal averages also expanded the perturbation fields, ω' and τ' , into the truncated Fourier series:

$$\omega' = \sum_{n=1}^{15} (A_n \sin n\lambda + B_n \cos n\lambda) \quad (9)$$

$$\tau' = \sum_{n=1}^{15} (A_n' \sin n\lambda + B_n' \cos n\lambda). \quad (10)$$

Thus each of the 21 latitude bands could have a maximum of 30 Fourier sine and cosine coefficients for the omega field and a similar number for the temperature field. A smaller number of non-zero coefficients for the more northerly latitude bands results because, due to the smaller circumference of these bands, insufficient grid points are present to define a wave of high number. The program was altered such that only the waves whose number was less than 1/5 the number of grid points within a latitude

band would produce non-zero Fourier coefficients. In this manner, if there were not at least five grid points for every wave around the latitude band, no wave of that number could be defined.

Having as initial data the Fourier coefficients of both the omega and the temperature fields, it was necessary to form the zonal average of the ω -perturbation and T -perturbation product, $[\omega' T']$, for each latitude band. The cross covariance of $\omega(\lambda)$ with $T(\lambda)$ is given by

$$\frac{1}{2\pi} \int_0^{2\pi} \omega(\lambda) T(\lambda) d\lambda = a_0 A_0 + \frac{1}{2} \left(\sum_{n=1}^{15} a_n A_n + b_n B_n \right). \quad (11)$$

By definition of the zonal average, the cross covariance of $\omega(\lambda)$ with $T(\lambda)$ is also equal to $[\omega(\lambda) T(\lambda)]$. Now inasmuch as

$$[\omega T] = [a T] = [a][T] \quad (12)$$

and

$$[T][T] = a_0 A_0, \quad (13)$$

it is evident that

$$[\omega T] = \frac{1}{2\pi} \sum_{n=1}^{15} a_n A_n + b_n B_n, \quad (14)$$

which is the cospectrum of $\omega(\lambda)$ and $T(\lambda)$. This quantity was computed for each latitude band, and then the meridional average of these 21 values was formed (i.e., $\{[\omega' T']\}$). Next multiplying this quantity by the appropriate constant (see equation (7)) yielded the rate of conversion from eddy potential energy to eddy kinetic energy.

Additionally, two other products were sought from the program. In order to ascertain the relative contribution of the 21 latitude bands to the eddy conversion, a print was made of $[\omega' T']$ for each band. Furthermore, to analyze the spectral components of eddy conversion and determine which wave numbers are most instrumental in effecting eddy

conversion, the Fourier coefficients products (i.e., $a_n A_n$ and $b_n B_n$) for each separate wave number were averaged over all 21 latitude bands.

These computations were carried out for each of the five surfaces. Ultimately for each time period analyzed there were produced one value of $C_{\omega,T}$, one value of $C_{z,T}$, one value of $C_{\bar{z},T}$, 21 acceptable values of $[\omega T']$, and 15 values representing the relative spectral components of $C_{\bar{z},T}$ for each of the five surfaces.

4. Results of computations

Table 1 shows the conversion of energy computed for 21 January 1962. The eddy conversion at all five levels is positive, indicating that eddy kinetic energy is being produced at the expense of eddy available potential energy, with a maximum eddy conversion in the vicinity of 700 mb. However, for the computations of this single day, it is found that the zonal conversion is from kinetic to potential and is greater in magnitude than the computed eddy conversion. No doubt, this is caused by the large positive zonal averages of omega in the southern bands on this day. Also note that the magnitude of the conversion due to mean sinking of the air mass is much larger than the magnitude of both the eddy and zonal conversion, unlike the results of the previous studies where the magnitude of the monthly space average of omega was not nearly so large.

Table 1. Energy conversion in the atmosphere north of 21N for 1200Z, 21 January 1962, in units of 10^{20} ergs mb⁻¹ sec⁻¹

Surface	1000 mb	850 mb	700 mb	500 mb	300 mb
$C_{\omega,T}$	-5.199	-6.904	-8.466	-8.541	-5.684
$C_{z,T}$	-0.143	-0.182	-0.230	-0.241	-0.169
$C_{\bar{z},T}$	+0.099	+0.108	+0.112	+0.101	+0.055

The relative contribution of the 21 latitude bands to the eddy conversion is presented in figure 2. As shown, the negative of the co-spectrum of omega and temperature, $-\overline{[\omega \theta]}$, and therefore the conversion from eddy potential energy to eddy kinetic energy, is greatest between 35N and 45N, with another relative maximum occurring in the vicinity of 70N.

Figure 3 indicates that for the day under consideration, wave numbers 1, 2, and 7 contribute significantly to the conversion from eddy potential energy to eddy kinetic energy at the four lower levels. At 300 mb, wave numbers 2 and 8 contribute most to the eddy conversion from potential to kinetic energy.

5. Conclusions

The computation of energy conversion at five levels in the atmosphere for this one day differs considerably from the pattern indicated by earlier investigations at only one pressure surface in the atmosphere, although table 1 indicates that calculation of conversion at the 600-mb surface would be quite representative of the entire atmosphere. However, if a legitimate comparison of the energy conversions is to be made, these computations must be extended to cover a period of at least one month. Now that the necessary initial fields and computer programs are available, the rapidity of the numerical computation makes the investigation of entire seasons feasible.

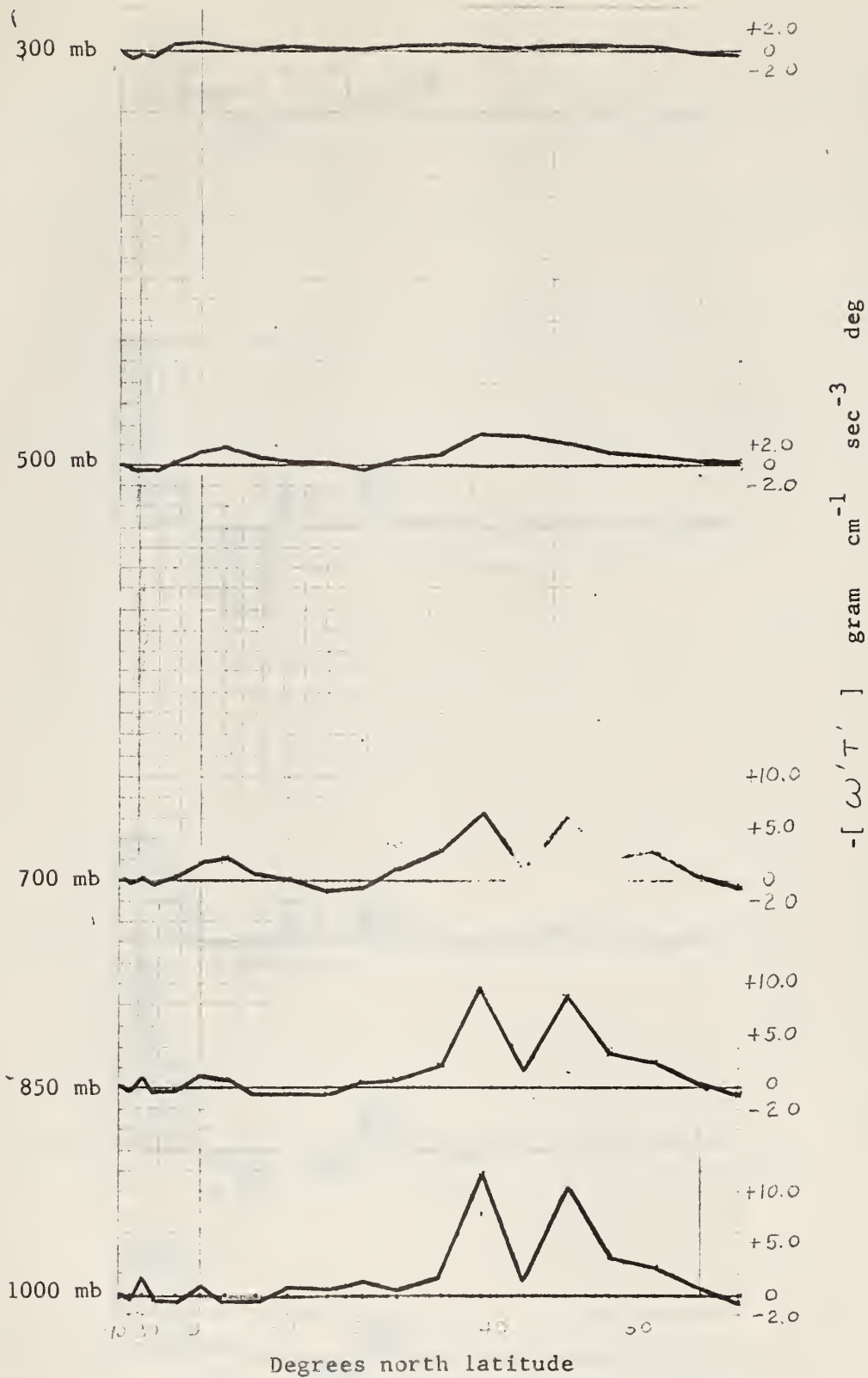


Figure 2. Negative cospectrum of omega and temperature, $-[\omega'T']$, for five surfaces at 1200Z, 21 January 1962.

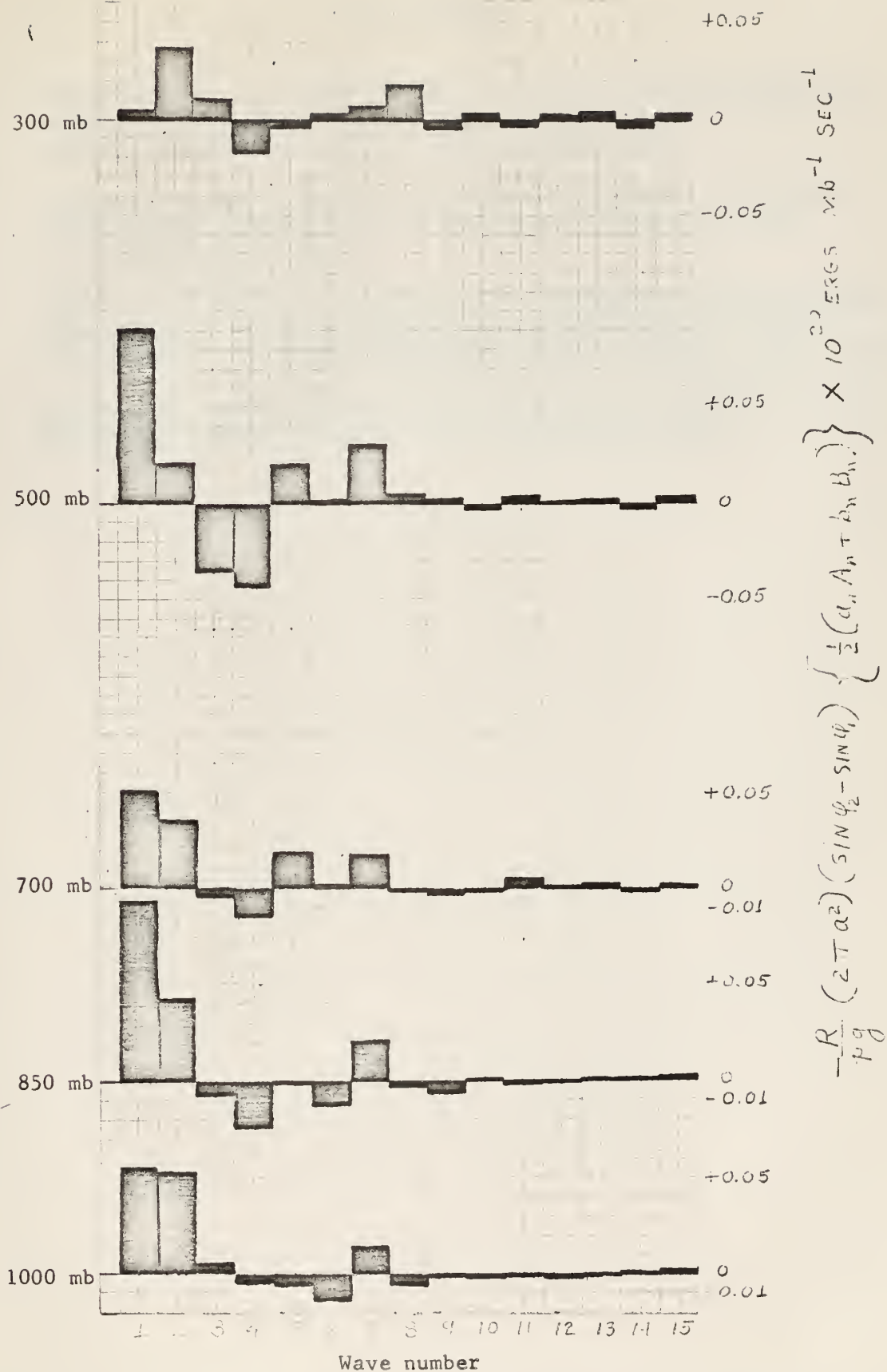


Figure 3. Spectral components of eddy conversion for five surfaces at 1200Z, 21 January 1962.

BIBLIOGRAPHY

1. Wiin-Nielsen, A., A study of energy conversion and meridional circulation for the large-scale motion in the atmosphere, Monthly Weather Review, 87 (9), pp 319-328, September 1959.
2. Saltzman, B., and A. Fleisher, The modes of release of available energy in the atmosphere, J. of Geophysical Research, 65 (4), pp 1215-1222, April 1960.
3. Clarke, L.C., and G. E. Lawniczak, Jr., Hemispheric solution of the omega equation including terrain and surface frictional effects, M.S. thesis, U. S. Naval Postgraduate School, Monterey, Calif., 1962.
4. Lorenz, E.N., Available potential energy and the maintenance of the general circulation, Tellus, 7 (2), pp 157-167, 1955.

DUDLEY KNOX LIBRARY



3 2768 00305700 1